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Thermal resistance of a multi-constrictions contact: A simple model

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Abstract

In this paper, a simple model is presented. It allows to give an approximation of the thermal contact resistance for a solid–solid contact showing several levels of defects (flatness, roughness, ...) and including therefore several scales in the constriction of the flux lines. According to this approach, the thermal contact is modeled using three intrinsic resistances (interstitial fluid resistance, resistance of asperities, constriction resistance). This allows a direct calculation, only valid for low levels of the ratio of the local contact over non-contact areas. Then the assumption is made that the constriction phenomena are independent from the resistance scale. Finally, a periodic distribution is considered for the surface defects.

1. Introduction

In a solid–solid contact, particularly due to the presence of flatness and roughness defects, the real surface of contact is not equal to the apparent one. In heat transfers, this phenomenon yields to a phenomenon called a heat flux constriction [1]. We can find in the literature a lot of contact resistance models in steady-state regime [2–5]. The goal of this paper is to present a simple thermal resistance model in steady-state regime that takes into account several levels of constrictions. This model can be called a "double constriction" model or "multi-constrictions" models. The references in this domain are poor. We can notice a paper by Warren [13] concerning the crushing of a Cantor type fractal surface with a fractal dimension varying from one to two and the paper of Majumdar [7] concerning a thermal model for a fractal contact. The geometry of the contact he considered is only of one type (a mono-pyramidal stack) and the resistance network does not take into account the interstitial fluid. Finally, we can mention the PhD thesis of Larzabal [6] in which the surface models he used are closed to those proposed in this paper and allow to model several types of surface. The thermal contact resistance is modeled by a network of resistances taking into account the constriction and the widening of the flux lines and the resistances of the asperities and of the interstitial fluid. In this paper, we will show by applying a similar approach that a simple model, in which the different resistances are intrinsic, can be set up in the

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Nomenclature

A, B, C, D	quadrupole coefficients
a	rod radius
b	asperity radius
l	rod length
N	number of constriction levels
r	radial coordinate
r_0	thermal resistance of the media in perfect contact
$r_a = R'/S_0$	thermal resistance of asperities associated to surface S_0
r_c	thermal contact resistance
r_{cf}	thermal constriction resistance associated to the outer flux tube
r_{cs}	thermal constriction resistance associated to the inner flux tube
$r_{ct} = X/(\pi b^2)$	intrinsic constriction resistance
r_f	thermal resistance of the fluid
$r_{fl} = R^*/S_1$	thermal resistance of the fluid associated to surface S_1
r_m	thermal resistance of the medium
$r_{mf} = l(\lambda S_1)$	thermal resistance of the medium associated to surface S_1
$r_{ms} = l/(\lambda S_0)$	thermal resistance of the medium associated to surface S_0
r_t	thermal resistance of the elementary cell
$R' = (\delta_1/\lambda_1) + (\delta_2/\lambda_2)$	thermal resistance of asperities per unit area
$R^* = (\delta_1 + \delta_2)/\lambda_f$	thermal resistance of fluid per unit area
s	real contact area
$s^* = b^2/a^2$	ratio of the real contact area over the elementary cell area
S	total contact area
S_0	inner flux tube area (see Fig. 2)
S_1	outer flux tube area (see Fig. 2)
T	temperature
T_0	imposed temperature
T_c	imposed temperature
X	intrinsic constriction resistance per unit area (see expression below)
$Y = (l_1/\lambda_1) + (l_2/\lambda_2)$	thermal resistance of medium 1 and 2 per unit area
z	axial coordinate
Greek symbols	
ε	similitude ratio
δ	asperity height
φ	heat flux density
φ_1	heat flux density through the asperity
φ_2	heat flux density through the fluid
ϕ	heat flux
λ	heat conductivity
Subscripts	
1, 2	related to medium 1 and medium 2
n	number of micro-contacts per macro-cell
Superscripts	
$*$	dimensionless quantity
m	related to the macro-cell

case of small asperities ($s^* \ll 1$) and allows a direct calculation of the contact resistance. To develop such a model, we are laying on a simplified model of the thermal contact resistance based on an elementary cell.

2. Model of an elementary cell

Whole studies are almost entirely based on the notion of a unit cell or an elementary cell that corresponds to a contact (for a given scale) associated to a heat flux tube. The geometry of the cell varies from one author to another [8] but cells are generally chosen similar with a periodic distribution. Some recent works [9–11] are interested in non-given dimensions cells with a random distribution. All these works clearly show that the results are less sensitive (less than 10%, expect to non-realistic particular cases 50%) to the cell geometry and the contacts distribution, for the same s/S (real contact area/total area) ratio and same asperities heights. On the other hand, the size distribution can have a non-negligible effect. Nevertheless, we will consider later on a classical model with a cylindrical geometry (see Figs. 1 and 2).

We mention here the assumptions for the calculation and refer the reader to [8,12] for more details:

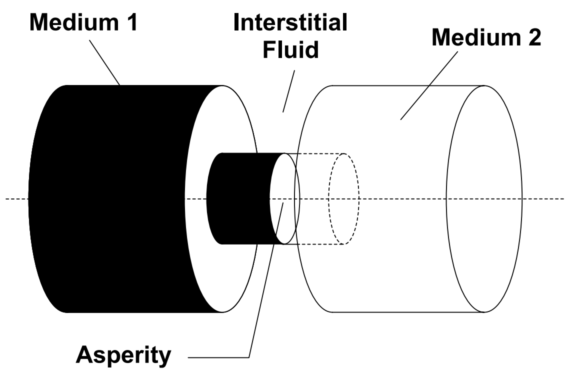

Fig. 1. Elementary cell.

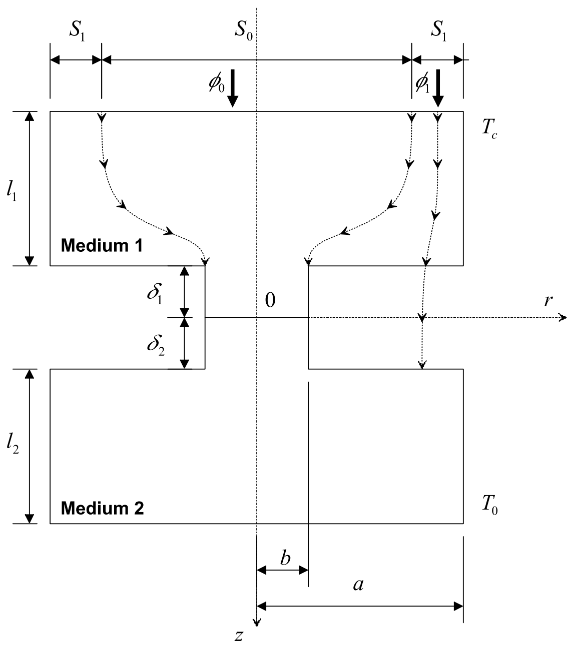

Fig. 2. Description of the elementary cell.

- Asperities heights δ_1 and δ_2 are weak compared to medium thicknesses l_1 and l_2.
- Boundary conditions between medium (1) and medium (2) are written in term of average temperatures over s and $S-s$.
- Heat flux is assumed piecewise uniform on s and $S-s$.

Under these conditions, the problem can be written with the following relations:

$$\begin{cases} A\varphi_0 + B\varphi_1 = T_c - T_0 \\ C\varphi_0 + D\varphi_1 = T_c - T_0 \end{cases} \quad (1)$$

with

$$A = Ys^* + R' + X$$
$$B = Y(1-s^*) - X$$
$$C = Ys^* - \frac{Xs^*}{1-s^*}$$
$$D = Y(1-s^*) + R^* + \frac{s^*X}{1-s^*}$$

where

- φ_0 and φ_1 represent the flux densities through the asperity and fluid respectively,
- T_c and T_0 are imposed temperatures to the system boundaries,
- $Y = \frac{l_1}{\lambda_1} + \frac{l_2}{\lambda_2}$: thermal resistance of medium 1 and 2 per unit area,
- $R' = \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2}$: thermal resistance of asperities per unit area,
- $R^* = \frac{\delta_1+\delta_2}{\lambda_f}$: thermal resistance of fluid per unit area,
- $X = \sum_{n=1}^{\infty} \frac{4J_1^2(\alpha_n b)}{\alpha_n^3 a^2 J_0^2(\alpha_n a)} \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)$: intrinsic constriction resistance per unit area (when l_1 and $l_2 \gg a$, practically l_1 and $l_2 > a$),
- $s^* = b^2/a^2$: ratio of the real contact area over the elementary cell area.

By defining the contact resistance as being the difference between the thermal resistance of the elementary cell (r_t) and the thermal resistance of media in perfect contact (r_0), we find:

$$r_t = r_c + r_0 \quad \text{that yields to} \quad r_c = \frac{T_c - T_0}{\varphi_0 \pi b^2 - \varphi_1 \pi (a^2 - b^2)} - \frac{Y}{\pi a^2}$$

Expressed per unit area:

$$R_c = \frac{R^*X(1-s^*) + XR's^* + R'R^*(1-s^*)}{R^*s^*(1-s^*) + R'(1-s^*)^2 + X} \quad (2)$$

Another approach consists in splitting the elementary cell into two parallel flux tubes. First one on the asperity (r_s) thrown by the flux ϕ_0 and the second on the fluid (r_f) thrown by the flux ϕ_1.

We can write

$$\frac{1}{r_t} = \frac{1}{r_s} + \frac{1}{r_f}$$

Let decompose each resistance r_s and r_f into four resistances connected in series:

$$\begin{cases} r_s = r_{ms1} + r_a + r_{cs} + r_{ms2} \\ r_f = r_{mf1} + r_{fl} + r_{cf} + r_{mf2} \end{cases}$$

where

- r_{ms1} and r_{ms2} represent the resistances of medium 1 and 2 associated to the surface S_0 (inner flux tube) ($r_{ms1} = l_1/(\lambda_1 S_0)$, $r_{ms2} = l_2/(\lambda_2 S_0)$),
- r_a and r_{fl} are the resistances of asperities and fluid ($r_a = R'/S_0$, $r_{fl} = R^*/S_1$),
- r_{mf1} and r_{mf2} represent the resistances of medium 1 and 2 associated to the surface S_1 (outer flux tube) ($r_{mf1} = l_1/(\lambda_1 S_1)$, $r_{mf2} = l_2/(\lambda_2 S_1)$),
- r_{cs} and r_{cf} are the constriction resistances associated to the inner flux tube (flux lines narrowing) and outer flux tube (flux lines widening).

This leads to the diagram of the Fig. 3. The expressions (1) allow us to show that potentials A_1 and B_1 are equal. So it is for potentials A_2 and B_2 (see Fig. 3). This result leads to the schema given in Fig. 4 and therefore to the expression of the contact resistance r_c like a set of four resistances:

$$\frac{1}{r_c} = \frac{1}{r_a + r_{cs}} + \frac{1}{r_{fl} + r_{cf}} \quad (3)$$

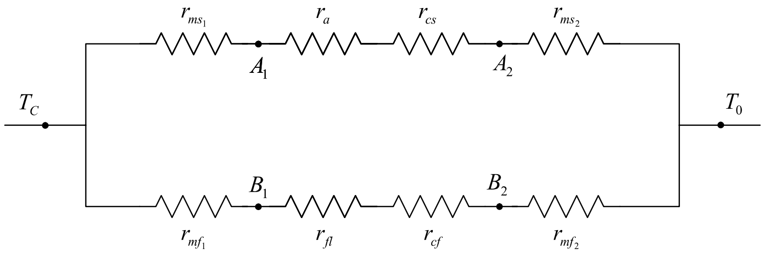

Fig. 3. Resistive diagram of the complete problem.

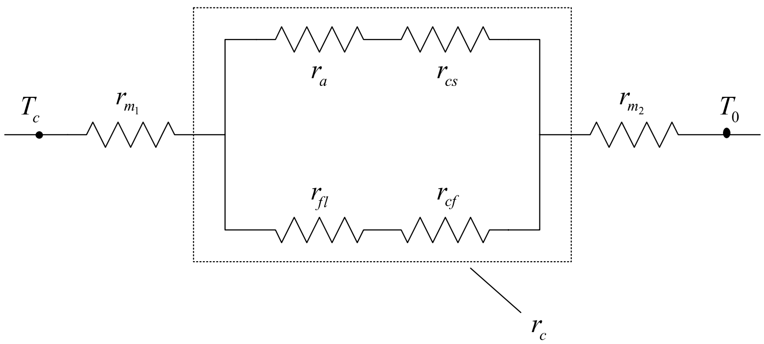

Fig. 4. Resistive diagram of the contact resistance.

The expressions of the constriction resistances r_{cs} and r_{cf} can then be determined from the relations (1) by identification:

$$\begin{cases} r_{cs} = \dfrac{X}{\pi b^2}\left(\dfrac{R^* - R'}{R^* + \frac{X}{1-s^*}}\right) \\ r_{cf} = \dfrac{X}{\pi(a^2 - b^2)}\dfrac{s^*}{1 - s^*}\left(\dfrac{R' - R^*}{R' + \frac{X}{1-s^*}}\right) \end{cases}$$

We can provide three comments on these two resistances:

- First, both are a function of R' and R^* and therefore are not intrinsic.
- Second, r_{cf} is negative, this express the widening of flux lines through the fluid.
- Finally, $|r_{cf}| \ll |r_{cs}|$ if b/a is weak $\left(\left|\dfrac{r_{cf}}{r_{cs}}\right| \text{ is in the same order of } \left(\dfrac{b}{a}\right)^4\right)$

This four resistances schema is exactly the same as those given by expression (2).

3. Three resistances simplified model

Due to the coupling between constriction resistances and asperities and fluid resistances, the previous schema is exact but difficult to use. Thus, we prefer to use a simplified model valid for weak b/a values ($s^* \ll 1$). That is almost always the case in usual applications.

For $s^* \ll 1$, relation (2) becomes

$$R_c = \frac{R^*\left(\dfrac{X}{s^*} + \dfrac{R'}{s^*}\right)}{R^* + \dfrac{X}{s^*} + \dfrac{R'}{s^*}} \tag{4}$$

That can be written (see Fig. 5):

$$\frac{1}{r_c} = \frac{1}{r_{fl}} + \frac{1}{r_{ct} + r_a} \tag{5}$$

We find again the fluid resistance r_{fl}, asperity resistance r_a and intrinsic constriction resistance r_{ct}

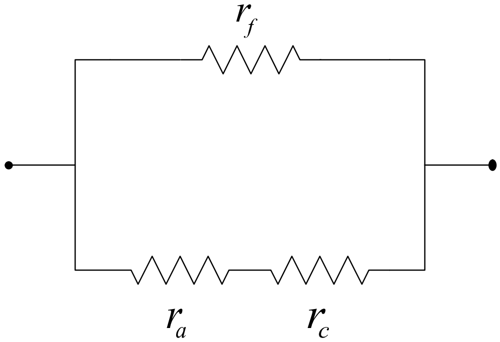

Fig. 5. Resistive diagram of the contact resistance.

with $r_{\mathrm{fl}} = \dfrac{R^*}{\pi a^2}$, $r_{\mathrm{a}} = \dfrac{R'}{\pi b^2}$ and $r_{\mathrm{ct}} = \dfrac{X}{\pi b^2}$

This is the representation that will be subsequently used.

4. Double constriction

At first, let consider the problem of a double constriction in which both flatness and roughness defects are taken into account. We can represent the contact as shown in Fig. 6 where the constrictions (macro and micro) are assumed independent that means in practice the distance δ_{m} is of the same order of magnitude than a (a_{m}, b_{m} and δ_{m} are the characteristics of the macro-cell and a, b, and δ the characteristics of the micro-cell).

In this case, the problem we have to solve in the macro-cell remains strictly the same as before, except in the level of the macro-asperities that now are not in perfect contact but divided by the micro-contact resistance that can be calculated by applying the previous result to a micro-cell. The equivalent electric schema is then given by the Fig. 7 and the contact resistance by the following expression:

$$\frac{1}{r_{\mathrm{c}}} = \frac{1}{\dfrac{1}{\left(\dfrac{1}{r_{\mathrm{fl}}} + \dfrac{1}{r_{\mathrm{a}} + r_{\mathrm{ct}}}\right) \cdot n} + r_{\mathrm{a_m}} + r_{\mathrm{ct_m}}} + \frac{1}{r_{\mathrm{fl_m}}} \tag{6}$$

where n is the number of micro-contacts by macro-cell. In fact, n represents the ratio of the macro-asperities area over the micro-cell area, that is $n = b_{\mathrm{m}}^2/a^2$ with the notations of the Figs. 2 and 6. By introducing the value of n and the notations of expression (4), the contact resistance per unit area is given by

$$R_{\mathrm{c}} = \frac{R_{\mathrm{m}}^*\left(\dfrac{X_{\mathrm{m}}}{s_{\mathrm{m}}^*} + \dfrac{R'_{\mathrm{m}}}{s_{\mathrm{m}}^*} + \dfrac{U}{s_{\mathrm{m}}^*}\right)}{R_{\mathrm{m}}^* + \dfrac{X_{\mathrm{m}}}{s_{\mathrm{m}}^*} + \dfrac{R'_{\mathrm{m}}}{s_{\mathrm{m}}^*} + \dfrac{U}{s_{\mathrm{m}}^*}} \tag{7}$$

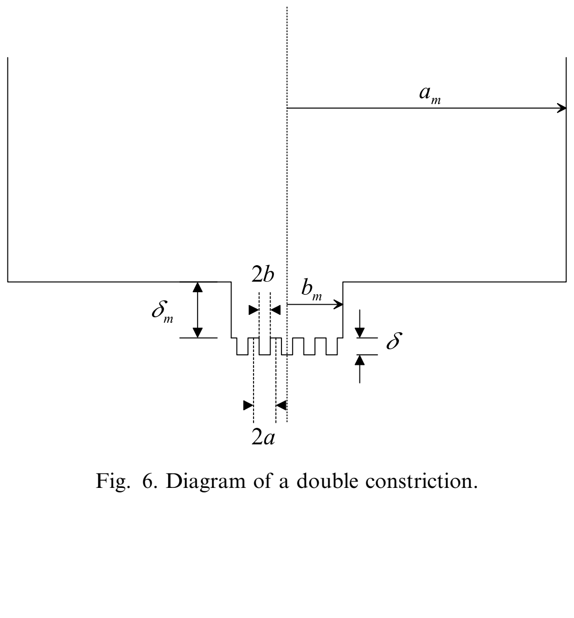

Fig. 6. Diagram of a double constriction.

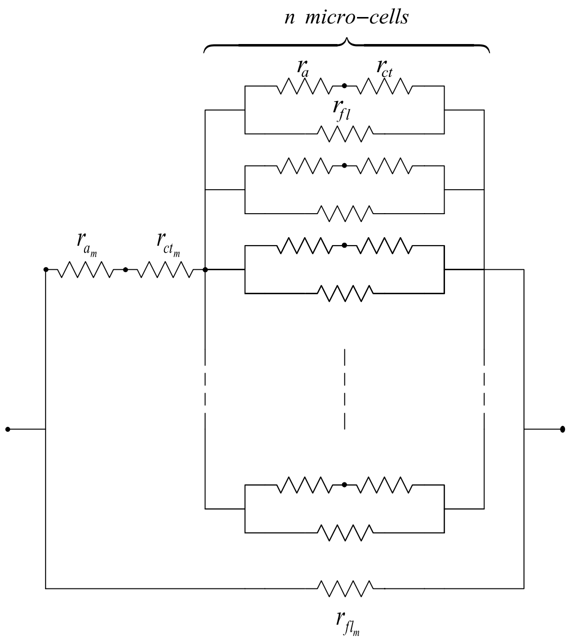

Fig. 7. Resistive diagram of the double constriction.

with

$$U = \frac{R^*\left(\dfrac{X}{s^*} + \dfrac{R'}{s^*}\right)}{R^* + \dfrac{X}{s^*} + \dfrac{R'}{s^*}}$$

(as a matter of fact, U is the micro-constriction resistance by micro-cell unit area) As shown in expression (7), in the more general case where heat flow through the fluid is taken into account, we cannot separate the contribution of the micro- and macro-contact resistances. Thus, it is difficult to draw some general conclusions from this expression.

For instance, let consider the following numerical result:

- Brass-Brass contact at 20 °C: $\lambda_1 = \lambda_2 = 110$ W m^{-1} K^{-1}
- Interstitial fluid. Air at 20 °C: $\lambda_{\mathrm{f}} = 0.025$ W m^{-1} K^{-1}.
- Microscopic parameters (roughness): $a = 300$ μm, $b = 30$ μm and $\delta = 1$ μm.
- Macroscopic parameters (flatness): $a_{\mathrm{m}} = 10$ mm, $b_{\mathrm{m}} = 1$ mm and $\delta_{\mathrm{m}} = 0.05$ mm.

This corresponds to the case where $n = 11$.

We obtain $X = 0.402 \times 10^{-6}$ m^2 K W^{-1}; $R^* = 0.800 \times 10^{-4}$ m^2 K W^{-1}; $R' = 0.182 \times 10^{-7}$ m^2 K W^{-1}; $X_{\mathrm{m}} = 0.134 \times 10^{-4}$ m^2 K W^{-1}; $R_{\mathrm{m}}^* = 0.400 \times 10^{-2}$ m^2 K W^{-1}; $R'_{\mathrm{m}} = 0.909 \times 10^{-6}$ m^2 K W^{-1}; $s_{\mathrm{m}}^* = s^* = 10^{-2}$; $U = 0.275 \times 10^{-4}$ m^2 K W^{-1} and $R_{\mathrm{c}} = 0.204 \times 10^{-2}$ m^2 K W^{-1}. Consequently, $r_{\mathrm{c}} = 6.49$ K W^{-1}.

If we calculate the macro-constriction alone (by neglecting the roughness), we find $r_c = 3.34\ \mathrm{K\,W^{-1}}$ and if we compare this result with the previous one, we observe that roughness defects doubled the resistance value. As a matter of fact, this result roughly depends on interstitial fluid that "short-circuits" more or less the micro-constriction resistance.

If the effect of the fluid is neglected ($\lambda_{fl} \to 0$), we find:

- For the double constriction $r_c = 17.9\ \mathrm{K\,W^{-1}}$.
- For the macro-constriction only $r_c = 4.55\ \mathrm{K\,W^{-1}}$.

The magnitude varies from 1 to 4. (Table 1 gives the r_c variations with respect to λ_f.) This first result seems us to be important because it shows that the effect of the fluid takes a larger part in the case of a double constriction model rather than in a simple constriction model. Let now consider the case where we can directly compare the part of the macro-constriction and micro-constriction resistances. To do this, we assume that heat flow through the fluid can be neglected. The resistance becomes

$$r_c = \frac{1}{\pi a_m^2}\left[\frac{X_m}{s_m^*} + \frac{R'_m}{s_m^*}\right] + \frac{1}{\pi b_m^2}\left[\frac{X}{s^*} + \frac{R'}{s^*}\right] = r_{c_{macro}} + r_{c_{micro}} \quad (8)$$

(indeed,

$$\frac{1}{\pi b_m^2}\left(\frac{X}{s^*} + \frac{R'}{s^*}\right) = \frac{1}{\pi a^2}\left(\frac{X}{s^*} + \frac{R'}{s^*}\right)\cdot\frac{1}{n}$$

represents the set of n in parallel micro-constrictions.)

To reduce the number of geometrical parameters that allow to describe the contact, we consider that we pass through macro- to micro-scales only by a simple change of scale (auto-similarity). (In the case of the example we considered, we should take $\delta_m = 33.3$ µm instead of 50 µm.)

In this case, the problem is defined by three dimensionless parameters and a length. For instance:

Table 1
Effect of the interstitial fluid conductivity on the micro-constrictions contribution

λ_f	r_{cd} double constriction	r_{cm} macro-constriction	r_{cd}/r_{cm}
0	17.9	4.55	3.93
0.001	16.7	4.49	3.72
0.01	10.5	3.98	2.64
0.1	2.34	1.87	1.25
1	0.300	0.297	1.01
10	0.0316	0.0316	1.00

$$\frac{a}{a_m} = \frac{b}{b_m} = \frac{\delta}{\delta_m} = \varepsilon \quad \text{similitude ratio}$$
$$\frac{b^2}{a^2} = \frac{b_m^2}{a_m^2} = s^* \quad \text{shape ratio} \quad (9)$$
$$\frac{\delta}{a} = \frac{\delta_m}{a_m} = \delta^* \quad \text{relative thickness of asperities}$$

For the length, we can choose a_m but we can generalized to another elementary cell shapes (for instance with rectangular or square bases) simply by replacing a_m by S, the area of the macro-cell (apparent contact area). That is $S = \pi a_m^2$ (see Ref. [8]).

Instead of ε, we could choose to use n, the number of micro-contacts by macro-cells; n being link to ε by (see Fig. 8):

Let write again the expression (8) (in the case of same materials in contact) by introducing the dimensionless parameters previously defined. We remember that to characterize the constriction X, we usually make appear a dimensionless parameter A which is only a function of s^* (see for instance [8,12]):

$$X = b\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)A(s^*) \quad (10)$$

with

$$A(s^*) = \frac{4}{\sqrt{s^*}} \sum_{n=1}^{\infty} \frac{J_1^2(\omega_n\sqrt{s^*})}{\omega_n^2 J_0^2(\omega_n)}$$

and ω_n being the roots of $J_1(\omega_n) = 0$.

Several approached expressions are given by authors. We will remember that for $s^* \ll 1$:

$$A(s^*) = 0.848(1 - 1.3\sqrt{s^*})$$

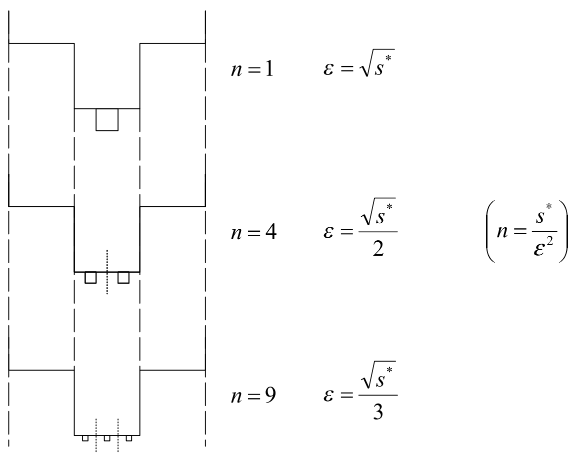

Fig. 8. Relation between n and the cell geometry (auto-similar square cells with double constriction).

In these conditions, we have

$$\begin{cases} R' = \dfrac{\delta_1}{\lambda_1} + \dfrac{\delta_2}{\lambda_2} = \dfrac{2\delta}{\lambda} \\ R'_m = \dfrac{2\delta_m}{\lambda} \\ X = b\left(\dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}\right) \cdot A(s^*) = \dfrac{2b}{\lambda} \cdot A(s^*) \\ X_m = \dfrac{2b_m}{\lambda} A(s^*) \end{cases}$$

And later on, expression (8) is written:

$$r_c = \frac{2}{\lambda\sqrt{\pi}\sqrt{S}}\left[\left(\frac{\sqrt{s^*}}{s^*}A(s^*) + \frac{\delta^*}{s^*}\right) + \frac{1}{s^*}\left(\frac{\sqrt{s^*}\varepsilon}{s^*}A(s^*) + \frac{\delta^*\varepsilon}{s^*}\right)\right] \tag{11}$$

By making appear a reduced resistance:

$$R_c^* = \frac{\lambda}{2} r_c \sqrt{\pi}\sqrt{S}$$

We obtain:

$$R_c^* = \left(\frac{A(s^*)}{\sqrt{s^*}} + \frac{\delta^*}{s^*}\right)\left(1 + \frac{\varepsilon}{s^*}\right) \tag{12}$$

Expression (12) particularly allows to compare the relative weight of the macro- and micro-scales. It is enough for that to compare ε/s^* with 1. If $\varepsilon > s^*$ then the micro-scale is more important than the macro-scale else if $\varepsilon < s^*$ then this is the opposite (we do not have to forget that ε is link to n by $n = s^*/\varepsilon^2$. So, n having to be larger than 1, ε cannot be larger than $\sqrt{s^*}$).

To conclude

$$\begin{array}{ll} 0 < \varepsilon < s^* & r_{c_{macro}} > r_{c_{micro}} \\ s^* < \varepsilon < \sqrt{s^*} & r_{c_{macro}} < r_{c_{micro}} \end{array}$$

We also have to come back on the notion of real surface of contact. This notion directly depends on the model that has been chosen to represent the interface and consequently has no physical sense. To illustrate this assertion, let consider three contacts with a double constriction and

$$\delta^* = 0, \quad s^* = 0.01 \quad \text{and} \quad \varepsilon = 0.1;\ 0.01;\ 0.001$$

Whatever the case, the real contact area is equal to

$s = s^{*2} \cdot S$ (non-dependent of ε)

Here,

$$s = 10^{-4} \cdot S$$

Let identify now each contact with a simple constriction model (as we usually do), and let calculate for the three previous cases the real area that should give the same contact resistance (to make the calculus more easy, let take $A(s^*) = A_0 = C^{ste}$).

$$R_c^* = \frac{A_0}{\sqrt{s^*}}\left(1 + \frac{\varepsilon}{s^*}\right)$$

Identified with a simple constriction model ($R_c^* = A_0/\sqrt{s^*}$) we find

$$s_1 = 0.83 \times 10^{-4} \cdot S$$
$$s_2 = 25 \times 10^{-4} \cdot S$$
$$s_3 = 83 \times 10^{-4} \cdot S$$

As a matter of fact, more generally $s = (s^*/(1 + (\varepsilon/s^*))^2) \cdot S$ (instead of the real area $s = s^{*2} \cdot S$) and strongly depends on ε because the identified value of s varies from 1 to 100.

5. Extension to a multi-constrictions surface

The diagram of the double constriction can be easily extended (under the same assumptions) to N-levels of constriction. Expression (7) becomes

$$U_i = \frac{R_i^*\left(\dfrac{X_i}{s_i^*} + \dfrac{R'_i}{s_i^*} + \dfrac{U_{i+1}}{s_i^*}\right)}{R_i^* + \dfrac{X_i}{s_i^*} + \dfrac{R'_i}{s_i^*} + \dfrac{U_{i+1}}{s_i^*}} \tag{13}$$

with $U_{N+1} = 0$ and $r_c = U_1/(\pi a_1^2)$, πa_1^2 being the macro-cell area.

Expression (13) is quite general. Its direct application is easy if we know the interface geometry. This requires the knowledge of three parameters for each level (a or S, b or s and δ). On the other hand, it is difficult to draw some generalities from this.

To analyze the expression (13), we consider the same conditions as for the double constriction (we will come back later on the influence of λ_f), that is

- $\lambda_f \to 0$
- levels are auto-similar.

This brings back the problem to three dimensionless parameters:

$$\begin{cases} \dfrac{a_i}{a_{i-1}} = \dfrac{b_i}{b_{i-1}} = \dfrac{\delta_i}{\delta_{i-1}} = \varepsilon \\ \dfrac{b_i^2}{a_i^2} = s^* \\ \dfrac{\delta_i}{a_i} = \delta^* \end{cases}$$

with $n = s^*/\varepsilon^2$.

Expression (13) can be simplified and becomes

$$\begin{aligned} r_c &= \frac{1}{\pi a_1^2}\left(\frac{X_1}{s^*} + \frac{R'_1}{s^*}\right) + \frac{1}{n}\cdot\frac{1}{\pi a_2^2}\cdot\left(\frac{X_2}{s^*} + \frac{R'_2}{s^*}\right) + \frac{1}{n}\cdot\frac{1}{n}\cdot\frac{1}{\pi a_3^2} \\ &\quad \cdot\left(\frac{X_3}{s^*} + \frac{R'_3}{s^*}\right) + \cdots + \left(\frac{1}{n}\right)^{N-1}\cdot\frac{1}{\pi a_N^2}\cdot\left(\frac{X_N}{s^*} + \frac{R'_N}{s^*}\right) \\ &= \sum_{i=1}^{N} \frac{1}{n^{i-1}}\cdot\frac{1}{\pi a_i^2}\cdot\left(\frac{X_i + R'_i}{s^*}\right) \end{aligned}$$

Let introduce the respective values of n, X_i and R'_i:

$$r_c = \sum_{i=1}^{N} \left(\frac{\varepsilon^2}{s^*}\right)^{i-1} \cdot \frac{1}{\pi a_i^2} \cdot \left[\frac{2b_i A(s^*)}{\lambda s^*} + \frac{2\delta_i}{\lambda s^*}\right]$$

Making appear the previous dimensionless parameters:

$$r_c = \frac{2}{\lambda\sqrt{\pi}\sqrt{S}} \cdot \left[\frac{A(s^*)}{\sqrt{s^*}} + \frac{\delta^*}{s^*}\right] \cdot \sum_{i=1}^{N} \left(\frac{\varepsilon}{s^*}\right)^{i-1}$$

Introducing the reduced resistance and calculating the sum, we find

$$R_c^* = \left(\frac{A(s^*)}{\sqrt{s^*}} + \frac{\delta^*}{s^*}\right) \cdot \frac{1 - \left(\frac{\varepsilon}{s^*}\right)^N}{1 - \frac{\varepsilon}{s^*}} \quad (\text{if } \varepsilon \neq s^*) \tag{14}$$

of which expression (12) is the particular case for $N = 2$.

This expression is interesting because it allows us to define two different behaviors according to the value of the similitude parameter ε.

If $\varepsilon > s^*$, R_c^* indefinitely increases with respect to N. This seams to be consistent with the fact that if $N \to \infty$ then the real contact area tends to zero ($s = s^{*N} \cdot S$).

If $\varepsilon < s^*$ then R_c^* is increasing with respect to N but tends to a limit value:

$$N \to \infty \quad R_c^* \to \left(\frac{A(s^*)}{\sqrt{s^*}} + \frac{\delta^*}{s^*}\right) \cdot \frac{1}{1 - \frac{\varepsilon}{s^*}}$$

This seams to be paradoxical because once again the real contact area tends to zero when $N \to \infty$ (s is non-dependant to ε). So, we can have a real contact area equal to zero and a finite contact resistance.

However, we do not have to lose sight of that all the previous results have been obtained in the limiting case where $\lambda_f = 0$. We have seen before that in the case of the double constriction, the values of the contact resistance can be strongly affected by the presence of interstitial fluid. This aspect is more amplifying for multi-constrictions.

By keeping auto-similar levels and introducing $\lambda^* = \lambda_f/\lambda$, expression (13) becomes

$$\frac{\lambda}{2} \cdot \frac{1}{a_i} \cdot U_i = \frac{\frac{\delta^*\varepsilon}{\lambda^*} \cdot \left[\sqrt{s^*}A(s^*) + \delta^* + \frac{\lambda}{2}\frac{1}{a_i}U_{i+1}\right]}{\frac{s^*\delta^*}{\lambda^*} + \sqrt{s^*}A(s^*) + \delta^* + \frac{\lambda}{2}\frac{1}{a_i}U_{i+1}}$$

Let

$$W_i = \frac{\lambda}{2}\frac{1}{a_i}U_i$$

We obtain

$$W_i = \frac{\frac{\delta^*\varepsilon}{\lambda^*} \cdot \left[\sqrt{s^*}A(s^*) + \delta^* + W_{i+1}\right]}{\frac{s^*\delta^*}{\lambda^*} + \sqrt{s^*}A(s^*) + \delta^* + W_{i+1}}$$

with $W_{N+1} = 0$ and $R_c^* = W_1/\varepsilon$.

Table 2
Reduced contact resistance (R_c^*) with respect to λ^* and N for $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.03$ ($n = 11$)

λ^*	N					
	1	2	3	4	10	∞
0	7.70	30.8	100.1	308	227 335	∞
10^{-5}	7.51	27.4	69.3	125	201	201
10^{-4}	6.13	13.9	18.7	20.4	21.1	21.1
10^{-3}	2.16	2.48	2.50	2.51	2.51	2.51
10^{-2}	0.29	0.29	0.29	0.29	0.29	0.29

Table 3
Reduced contact resistance (R_c^*) with respect to λ^* and N for $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.01$ ($n = 100$)

λ^*	N					
	1	2	3	4	10	∞
0	7.70	15.4	23.1	30.8	77	∞
10^{-5}	7.51	14.5	20.6	25.9	41.1	44.4
10^{-4}	6.13	9.46	10.9	11.5	11.8	11.8
10^{-3}	2.16	2.30	2.31	2.31	2.31	2.31
10^{-2}	0.29	0.29	0.29	0.29	0.29	0.29

Table 4
Reduced contact resistance (R_c^*) with respect to λ^* and N for $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.003$ ($n = 1111$)

λ^*	N					
	1	2	3	4	10	∞
0	7.70	10.0	10.7	10.9	11	11
10^{-5}	7.51	9.63	10.2	10.4	10.5	10.5
10^{-4}	6.13	7.24	7.43	7.46	7.47	7.47
10^{-3}	2.16	2.21	2.21	2.21	2.21	2.21
10^{-2}	0.29	0.29	0.29	0.29	0.29	0.29

For instance, Table 2 shows the evolution of R_c^* with respect to λ^* for several N values in the case $\varepsilon > s^*$ for $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.03$. (This involves $A(s^*) = 0.74$.)

We actually notice that the influence of the fluid is more important when N is large. Especially for $N = 4$, the presence of interstitial fluid cannot be neglected even in the case of a low pressure gas.

Tables 3 and 4 present the results we respectively obtain in the cases $\varepsilon = s^*$ (for $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.01$) and $\varepsilon < s^*$ (with $\delta^* = 0.003$; $s^* = 0.01$ and $\varepsilon = 0.003$).

6. Fractal dimension

It can be interesting to have a look on the fractal dimension of the surface of contact. By using the HAUSDORFF dimension [14] in auto-similar cases, D is given by

$$D = -\frac{\ln[N(l)/N(l')]}{\ln[l/l']} \quad (15)$$

(l and l' being the diameters of the elements between two succeeding levels $N(l)$ and $N(l')$ the number of elements between two succeeding levels).

This yields with our notation to the following relation:

$$D = \frac{\ln(s^*/\varepsilon^2)}{\ln(1/\varepsilon)} \quad (16)$$

- If $s^* = 1$, we find $D = 2$. This is an evident result.
- If $\varepsilon \to 0$, we also find $D = 2$. This is less obvious.
- The limit as $\varepsilon = s^*$ which is the limiting case between the two behaviors of the contact resistance corresponds to a fractal dimension equal to $D_l = 1$.
- If $\varepsilon \to \sqrt{s^*}$, we find $D = 0$.
- For instance, the fractal dimensions in the three cases we considered in Tables 2–4 are

$D_{\mathrm{II}} = 0.687$
$D_{\mathrm{III}} = 1$
$D_{\mathrm{IV}} = 1.207$

7. Conclusion

A simple model for the thermal behavior of a multi-constrictions contact has been developed from a contact resistance diagram composed of a set of three resistances.

This elementary model nevertheless allows us to underline

- the relative importance of macro- and micro-constrictions conducted by the ratio ε/s^* (similitude ratio over shape ratio),
- a behavior of the n-constrictions contact that strongly depends on the ratio ε/s^*,
- the key role played by the interstitial fluid when the order N becomes large,
- the non-intrinsic character of the notion of real contact area.

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